

**Timber Appraisal Zones and Timber Values:
A Report to the Montana Department of Revenue**

Revised
FINAL REPORT

by

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I. Introduction

The purpose of this report is to revalidate the boundaries of the forestland tax appraisal zones in Montana, and provide an updated average stumpage price for each appraisal zone. The basic methodology of this study was presented in Jackson (1992, 1996). This report is an extension of that methodology. Based upon the analysis presented below, the boundaries of the tax appraisal zones have not changed from those presented in the 1996 report. However, timber prices have changed.

II. Relation to Previous Appraisal Estimates

As was the case of the previous appraisal analysis, state lands timber sales are used to determine the appraisal zones and average timber prices using methods of transactions evidence appraisals documented originally in Jackson and McQuillan (1979).

Two databases are used for the analysis. The transaction evidence appraisal database used by the Division of Forestry, Department of Natural Resource (DNRC) is utilized. Original data was collected and is utilized for DNRC sales in eastern Montana since the state does not follow the same appraisal procedures in selling timber in the eastern portion of the state. The database includes sales for state fiscal years 1997 through 2001. The breakdown of sales is summarized in Table 1 (page 5). The total number of transactions used in the study (115) represents an increase in observations from the previous study (82 transactions). The increase in observation results mainly from utilizing a 5-year database. A 3-year period was used in the previous study. Therefore, this database is more adequate than had been the case in earlier studies.

A new issue is confronted in this analysis. The Forestry Division of the Montana Department of Natural Resources initiated a policy change concerning the physical unit of measure for each timber sale. Previously, all timber was sold, measured (scaled), priced and sold in terms of the Scribner board foot measure. Furthermore, the Montana DOR's productivity classes are measured in cubic volume. In past appraisal cycles, cubic volume was converted to board feet and value used the same common denominator. During the 5-year study period the Forestry Division started to phase out the Scribner measure and use a weight measure (tons) for pricing. The weight scale measure was converted by the DNRC from cruise saw timber volumes using a variety of conversion ratios. Some of the transactions in the study are sold and priced using the Scribner measure and other transactions are sold and measured using the ton measure.

The relationship between the Scribner measure and the weight measure is not perfectly well behaved. Scribner is a volume measure as opposed to a weight measure. The ratio of Scribner measure per ton no doubt depends on a variety of factors. These include moisture content of logs, species, and log sizes. It is fair to hypothesize that the unit used to measure logs removed will affect the actual amount bid. A considerable effort went in to examining how to remove any bias in bidding that might result from the substitution of the weight measure of wood for the earlier volume measure. If the Montana Department of Natural Resources and Conservation eventually converts to tonnage as the sole measure for selling timber, then the DOR should develop conversion factors for converting cubic volume productivity estimates to a tonnage basis for future appraisal cycles.

In the earlier studies, transaction data was grouped into two samples. This was also done in this study. One sample represents transactions for four appraisal zones in the western portion of the state and the other sample represents transactions for sales in remaining eastern zone. Actually, this two-sample approach to estimating zones and prices is necessitated in part by the differences in the way the DNRC appraises timber in the eastern portion of Montana. Unlike sales in the rest of the state, there is no estimated cost of purchaser road construction or estimated selling price lumber tally associated with the timber being auctioned. Furthermore, there appears to be substantial differences in contractual conventions for brush disposal, reforestation and timber stand improvement funds in the eastern sales from the other DNRC transactions. Since some of the key analytical variables are defined differently for the eastern sales, it is useful to separate the transactions into two separate pools.

III. Revised Timber Appraisal Zones

Figure 1 represents a map of the timber tax appraisal zones. Identification of the counties along with the number of sales transactions within each county for the appraisal period is presented in table 1.

Table 1 ...
Timber Tax Regions in Montana

Northwest Montana--Zone 1 Counties--(Number of Transactions)			
	Flathead	(19)	
	Lake	(6)	
	Lincoln	(9)	
	Sanders	(10)	
			subtotal 44
West Central Montana--Zone 2			

	Mineral	(4)	
	Missoula	(8)	
	Powell	(6)	
	Ravalli	(6)	subtotal 24
South Central Montana--Zone 3			
	Beaverhead	(3)	
	Jefferson	(2)	
	Lewis and Clark	(5)	
	Madison	(1)	subtotal 11
Central Montana--Zone 4			
	Cascade	(1)	
	Gallatin	(3)	
	Meagher	(2)	subtotal 6
Eastern Montana--Zone 5			
	Big Horn	(2)	
	Carter	(3)	
	Custer	(1)	
	Fergus	(8)	
	Musselshell	(8)	
	Powder River	(5)	
	Sweet Grass	(1)	
	Treasure	(2)	<u>subtotal 30</u>
	Total Population		115

IV. Updated Regression Models

The two regression models that are used to link the clustering of county groupings into an appraisal zone and to provide the basis for predicting the average stumpage price for each county grouping are presented below. As was the case before, alternative county configurations were examined prior to arriving at the final grouping shown above in table 1.

Model 1 in Table 2 is for the appraisal Zones 1 through 4, and Model 2 in table 3 is for Appraisal Zone 5 data.

The dependant variable in Model 1 is defined as the real adjusted winning bid. It

Table 2--Model 1
Predicting Timber Prices in Zones 1, 2, 3, and 4

Ordinary least squares regression

Dependent variable is Y1 Mean = 196.43336, S.D. = 131.8379
 Model size: Observations = 85, Parameters = 13, Deg.Fr. = 72
 Residuals: Sum of squares= 110443. Std.Dev. = 39.16537
 Fit: R-squared = 0.92436, Adjusted R-squared = 0.91175
 Model test: F[12, 72] = 73.32, Prob value = 0.00000
 Breusch - Pagan chi-squared = 25.2357, with 12 degrees of freedom

Variable	Coefficient	Standard Error	t-ratio	P-Value
Constant	187.44	50.383	3.720	0.00039
REALSPLT	0.33693	0.13999	2.407	0.01866
RSPLT	-0.66802	0.83380E-01	-8.012	0.00000
VOLSOLD	3.6151	2.1071	1.716	0.09052
TONVOL	-0.31439	0.48398	-0.650	0.51803
LOC1	63.699	20.950	3.040	0.00329
NWTONS	-74.586	26.087	-2.859	0.00555
LOC2	36.957	22.365	1.652	0.10279
WCENTONS	-43.661	27.784	-1.571	0.12047
LOC4	24.836	21.893	1.134	0.26039
CENTONS	-17.072	34.719	-0.492	0.62442
LOGSYS	-65.016	10.074	-6.454	0.00000
LOGSTONS	40.350	15.584	2.589	0.01163

is the sum of the purchasers winning bid plus the required payments of earmarked funds for slash disposal and timber stand improvements, along with the estimated road construction and road maintenance expressed in constant 1996 dollars. When the sale is a board foot sale, Y1 is expressed in dollars per thousand board feet. When the sale is a ton sale the adjusted winning bid is expressed in dollars per ton. The equation summarized in Figure 1 is significant based on the calculated F and explains about 91% of the total variation in the winning bid prices. The independent variables reflect the

utilization of a dummy variable (a variable that takes on the value of zero or one) and sale characteristic variables. The sale characteristics include REALSPLT, the real selling price of the estimated lumber content of the sale. This variable reflects of lumber prices of the various wood species included in the sale. The volume sold (VOLSOLD) is the volume in million board feet. Even when the sale is a ton sale an estimated board foot volume is included in this variable. TONVOL is the estimated tonnage for ton sales. Otherwise the value of this variable is zero. Location variables are also included. LOC1 indicates the sale is in the Northwest Appraisal zone. LOC2 is the West Central Appraisal zone and LOC4 is the Central Appraisal zone. LOGSYS is a logging cost proxy variable and is calculated by applying different respective weights to the proportion of each sale logged with tractors, ground lead and helicopters. It is equal to:

$$\text{LOGSYS} = 1 * \text{Prop Tractor} + 1.5 * \text{Prop Ground Lead} + 2 * \text{Prop Helicopter}$$

All of the other variables in the above equation reflect the use dummy variables. RSPLT is equal to REALSPLT for ton sales, otherwise it is zero. NWTONS is 1 for ton sales in zone 1, otherwise it is zero. The same is true for the other independent variables in the equation (WCENTONS, CENTONS, LOGSTONS). WCENTONS has a value of 1 for west central zone ton sales, otherwise it is zero. Similarly, CENTONS is one for central zone ton sale and zero otherwise. LOGSTONS is equal to LOGSYS for ton sales and equal to zero for the rest.

Model 1 was also evaluated for heteroskedasticity. When a statistical model has heteroskedasticity, the residual (or difference between the predicted amount and the actual amount) is functionally related to the size of the dependant variable. Only the

corrected version of the model based upon a corrected covariance matrix is presented in table 1.

The p-values indicate the probability that each coefficient is non-zero. All coefficients are significant at an alpha level of .05 using two-tailed t-tests. With the exception of LOC2, WCENTONS, LOC4 and CENTONS, all of the independent variables are significant at an alpha level of .05. (Note that while Loc2 and Loc4 are not significantly different from zero on a two-tailed test, they are using a one-tailed test of significance. They are also spatially separated by zone 3.)

The model predicting the bid values for Eastern Montana (Zone 5) sales is summarized in Table 2. Based on the calculated level of F, the overall model is significant. Considering the less precise or even nonexistent appraisal system used on Eastern Montana sales, the predictive power of the equation is quite good. The R-square adjusted for the number of degrees of freedom is 0.77089. Seventy-eight percent of the total variation in price is explained by the equation. The definition of each of the independent variables in model 2 is the same as is the case of model 1. Model 2 is also corrected for heteroskedasticity.

Table 2-Model 2

Predicting Eastern Montana Timber Prices

Ordinary least squares regression

Dependent variable is Y1 Mean = 88.80291, S.D. = 66.4155

Model size: Observations = 30, Parameters = 5, Deg.Fr. = 25

Residuals: Sum of squares= 25265.1 Std.Dev. = 31.78998

Fit: R-squared = 0.80249, Adjusted R-squared = 0.77089

Model test: F[4, 25] = 25.39, Prob value = 0.00000

Results Corrected for heteroskedasticity

Breusch - Pagan chi-squared = 13.4328, with 4 degrees of freedom

Variable	Coefficient	Standard Error	t-ratio	P-Value
REALSPLT	0.33868	0.23307E-01	14.532	0.00000
RSPLT	-0.37231	0.32894E-01	-11.318	0.00000
TONVOL	1.6158	0.70719	2.285	0.03107
TONS	27.756	10.427	2.662	0.01338
VOLSOLD	-5.1761	3.0114	-1.719	0.09800

V. Summary Statistics of the Appraisal Zones

From the standpoint of further analyzing the differences in timber prices between zones, a set of summary statistics are introduced for sales in each zone. The arithmetic

TABLE 3

DESCRIPTIVE STATISTICS OF THE VARIABLES

Zone 1						
Variable	Mean	Std. Dev.	Skew.	Kurt.	Minimum	Maximum
REALSPLT	368.5576	48.0162	0.5	4.4	260.5627	523.4530
RSPLT	161.7768	172.8851	0.1	1.1	0.0000	396.5810
VOLSOLD	2.1909	1.7222	1.7	6.6	0.1760	8.9000
RELDECST	34.3586	28.1602	1.0	3.4	0.0000	115.0299
RELDMCT	3.7425	6.4298	3.5	17.6	0.0000	36.2734
TONVOL	5.4912	8.6208	1.4	3.5	0.0000	29.8285
LOGSYS	1.1673	0.3992	3.6	16.3	1.0000	3.1000
LOGSTONS	0.5253	0.5672	0.2	1.3	0.0000	1.6400

Zone 2						
Variable	Mean	Std. Dev.	Skew.	Kurt.	Minimum	Maximum
REALSPLT	373.9297	40.6290	1.1	3.2	322.6800	479.1990
RSPLT	106.1662	169.6942	0.9	1.9	0.0000	426.6220
VOLSOLD	2.0689	1.8661	1.7	5.3	0.1970	7.9040
TONVOL	2.1420	5.6205	3.4	14.4	0.0000	26.0910
RELDECST	36.7179	28.3610	1.3	4.4	2.3856	119.4641
RELDMCT	3.5396	2.8173	0.9	2.9	0.0000	10.0126
LOGSYS	1.0966	0.1926	2.0	5.3	1.0000	1.6400
LOGSTONS	0.3233	0.5289	1.1	2.6	0.0000	1.6400

Zone 3

<u>Variable</u>	<u>Mean</u>	<u>Std. Dev.</u>	<u>Skew.</u>	<u>Kurt.</u>	<u>Minimum</u>	<u>Maximum</u>
REALSPLT	375.0677	44.8315	1.2	3.1	330.2675	475.1600
RSPLT	94.1139	161.2920	1.0	1.9	0.0000	353.8950
VOLSOLD	1.7517	1.9049	1.3	3.1	0.2320	5.7915
TONVOL	3.5211	9.9264	2.6	8.0	0.0000	33.1500
RELDECST	30.9286	18.3411	0.9	2.4	14.1901	69.0220
RELDMCT	2.3152	4.7181	2.3	6.8	1.0000	3.1000
LOGSYS	1.2783	0.6363	2.3	6.8	1.0000	3.1000
LOGSTONS	0.4914	0.9827	1.8	5.1	0.0000	3.1000

Zone 4

<u>Variable</u>	<u>Mean</u>	<u>Std. Dev.</u>	<u>Skew.</u>	<u>Kurt.</u>	<u>Minimum</u>	<u>Maximum</u>
REALSPLT	397.3947	61.5770	0.1	1.3	321.5740	481.2570
RSPLT	80.2095	196.4723	1.6	3.5	0.0000	481.2570
VOLSOLD	0.8245	0.7478	1.4	3.1	0.2500	2.3010
TONVOL	2.4928	6.1060	1.6	3.5	0.0000	14.9565
RELRECST	38.5980	18.1371	-0.7	1.9	10.0394	57.6631
RELREDCT	0.0000	0.0000	0.0	0.0	0.0000	0.0000
LOGSYS	1.1330	0.2179	0.9	2.0	1.0000	1.5110
LOGSTONS	0.1667	0.4082	1.6	3.5	0.0000	1.0000

Zone 5

<u>Variable</u>	<u>Mean</u>	<u>Std. Dev.</u>	<u>Skew.</u>	<u>Kurt.</u>	<u>Minimum</u>	<u>Maximum</u>
REALSPLT	396.3490	61.0266	0.3	3.0	265.4065	512.7230
RSPLT	151.0256	191.3777	0.5	1.4	0.0000	503.8170
VOLSOLD	0.8827	0.9868	3.3	15.6	0.1690	5.4250
TONVOL	2.4861	3.6478	1.1	2.7	0.0000	10.8100
RELDECST	26.5332	40.7299	1.0	1.7	1.3405	87.1545 n=4
RELDMCT	0.0000	0.0000	0.0	0.0	0.0000	0.0000
LOGSYS	1.0516	0.2828	5.1	27.1	1.0000	2.5490
LOGSTONS	0.4000	0.4983	0.4	1.1	0.0000	1.0000

Means for the independent variables for each zone are used to predict the representative price for each zone during the appraisal period.

VI. Reconciling Timber Prices in the Appraisal Zones

Recall that the dependent variable is defined to include the winning bid plus development costs, earmarked funds and road maintenance costs. Because all sales are adjusted to constant 1996 dollars using the Implicit GDP Deflator, sample means are used to estimate the average price for each zone in real adjusted price dollars. Then the average costs of development and road maintenance are subtracted from the predicted adjusted average price for each zone. This is shown in table 3. Rather clearly, average timber prices do differ between the appraisal zones. Furthermore, in the sense of identifying areas of homogeneity of price or value, the model in table 2 indicates that the spatial patterns of value are "well behaved" or "predictable". Variation in timber prices is explained by variations in qualitative characteristics of the timber sale and the location of the timber being sold. While the average real adjusted winning bid will be used to reconcile the price for each zone, zone prices were also calculated using the sample means for the independent variables from each zone.

First the prices will be reconciled to the basis of what the timber would sell for if all of the roads were constructed as a logging expense. In doing this, average road maintenance costs (REL RDMCT) and average development costs (REL DECST) in each zone are subtracted from the inflation adjusted real winning bid (REL ADBID) for Zones 1 through 4. The difference between the real adjusted winning bid and the subtracted road maintenance costs and development costs represents the payment for timber, and earmarked funds for investment in the next timber crop.

Table 4
Prices Per Appraisal Zone (1996 Dollars)

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
RELADBID	\$307.34	\$286.57	\$237.03	\$275.49	\$129.67
Less Road Maint.					
And Dev Costs	<u>38.10</u>	<u>40.26</u>	<u>33.24</u>	<u>38.59</u>	<u>3.42</u>
Price (1996\$)	\$269.73	\$246.31	\$203.79	\$236.90	\$125.91

The above prices are expressed in constant 1996 dollars. For tax purposes they are to be converted to 4th quarter 2001 dollars. At this time the 4th quarter price index has not been released so a "preliminary" estimate is used so that the prices can be adjusted to the published index when it becomes available. A preliminary index of 1.10 is used in this draft.

Table 5
Prices Per Appraisal Zone
(4th Quarter 2001 dollars)

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
Price	\$269.73	\$246.31	\$203.79	\$236.90	\$125.91
Adj. Factor	<u>*1.0986</u>	<u>*1.0986</u>	<u>*1.0986</u>	<u>*1.0986</u>	<u>*1.0986</u>
Adj. Price	\$296.33	\$270.60	\$223.88	\$260.26	\$138.32

VII. Changes in Prices from the Previous Appraisal Period

Table 5 below shows the differences in timber prices for the five appraisal zones between the current versus the previous two appraisal periods. The previous prices were developed by Jackson (1992, 1996) and are reported in the dollars used for actual appraisal.

Table 5

Updated and Previous Appraised Prices

	1993 Price	1996 Price	2001 Price
Zone 1	\$212.39	\$371.87	\$296.33
Zone 2	166.66	320.91	\$270.60
Zone 3	112.32	208.89	\$223.88
Zone 4	134.65	264.65	\$260.26
Zone 5	46.18	137.17	\$138.32

The previous prices were based on timber transactions for the fiscal years 1988 through 1992 and the prices were expressed in 1992 dollars. It is apparent from Table 5 that timber prices have only increased modestly in Zone 3. The price in Zone 5 is virtually the same as was the 1996 price. The prices in Zones 1, 2, have dropped considerably from their 1996 levels. The drop in Zone 4 is more modest. The price in Zone 5 is virtually the same while the price in zone 3 has increased about 7% from the 1996 appraisal cycle.

The prices in Zones 1 through 4 seem somewhat more homogeneous than was the case in previous appraisal cycles.

VIII. Conclusions

Based upon a set of timber transactions and the examination of county groupings. The statistical models are significant, as are the spatial groupings of contiguous counties. The statistical models perform in accordance with earlier published literature regarding transactions evidence appraisals. Timber prices are substantially lower in two appraisal zones. Changes in price for two other zones are quite small. Only one zone experienced a price increase from the previous appraisal period.

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Appendix

Stumpage Valuation with Board Foot and Ton Sales

The advent of DNRC sales sold and scaled in tons rather than Scribner board feet necessitated the use of “dummy variables as a jack knife” procedures in the transaction evidence appraisal equations. This appendix will explain in more detail the equations presented in the main body of the report. The dummy variables as a jack knife allows the use of the entire sample population to in effect estimate two equations, one for board feet sales and one for ton sales.

Sales sold and scaled by the ton did not occur randomly. For example, all of the west side sales used in the Westside model were sold and scaled by the board foot from January, 1995 through May of 1998. During the period of June 1998 through July 1999, the DNRC sold both ton and board foot sales. From August 1999 through June of 2000, all west side sales were sold and scaled by the board foot. A similar pattern of sales emerged in the east side appraisal zone.

On average the overall market conditions in which ton sales occurred were lower than was the case when board foot sales occurred. This necessary meant that ton sales could not be excluded from the population as that process would have resulted in unfairly high timber prices and timber taxes

One is tempted to use a “conversion factor” to convert the ton volumes to board feet as well as the bid prices. There are a variety of pitfalls that rule out such a simplistic approach. First, no good conversion factor is readily available for species, location and time of year. Second, bidders have had to develop proprietary information on how ton sales actually convert to finished products and this information is not publicly available. Third, if ton sales are an inferior marketing measure to board feet, bidders may down bid tons sales relative to their board foot equivalents.

Thus, two realities that affected the approach to developing appraisal equations emerged. First dropping the ton sales from the sample wasn't consistent with the estimation of fair market values. Second, use of an ad hoc set of ton to board feet conversion factors was also inconsistent with fair appraisal methods. As a result, the use of dummy variables was used. This approach is consistent with using market information to reveal how bidders adjust their bids to tons or board feet. It allows the use of the entire sample population and as it turns out, gives quite a robust set of transactions evidence appraisal equations.

Dummy variables take on the value of zero or one. Thus, all ton sales were coded 1 and non-ton (board foot) sales were coded zero. The east side equation is used first to show how the model applies to both ton and board foot sales simply because there are fewer (5) independent variables and it is easier to use for demonstration purposes. The decomposition of the west side equation is conceptually the same but the 13 variables make it a bit clumsier to demonstrate. It is done for the northwest zone of the west side. The other regions are not decomposed simply for expedience.

The East-Side Equation Combined Tons and Board Feet

$$\text{Real Adjusted Winning Bid} = 0.33868 \cdot \text{REALSPLT} - 0.37231 \cdot \text{RSPLT} + 1.6158 \cdot \text{TONVOL} + 27.756 \cdot \text{TONS} - 5.1761 \cdot \text{VOLSOLD}$$

By setting the value of the dummy variable to 0 indicating that the sale is a board foot sale, the equation becomes the following

$$\text{Real Adjusted Winning Bid} = 0.33868 \cdot \text{REALSPLT} - 5.1761 \cdot \text{VOLSOLD}$$

And the predicted bid is in dollars per thousand board foot.

By setting the value of the dummy variable to 1, the equation becomes:

$$\begin{aligned} \text{Real Adjusted Winning bid} &= 0.33868 \cdot \text{REALSPLT} - 0.37231 \cdot \text{REALSPLT}/6.5 - \\ &1.5158 \cdot \text{TONVOL} + 27.756 \cdot \text{TONS} - 5.1761 \cdot \text{VOLSOLD} \\ &\text{because RSPLT} = \text{REALSPLT}/6.5 \text{ for ton sales.} \end{aligned}$$

This Ton Sale equation can be simplified to the following:

$$\text{Real Adjusted Winning Bid} = 27.756 + 0.2814 \cdot \text{REALSPLT} + 1.6158 \cdot \text{TONVOL} - 5.1761 \cdot \text{VOLSOLD}$$

The constant term of 6.5 was used initially to convert the Real Selling Price Lumber Tally to an approximate ton equivalent. Since the coefficient (-.37231) is clearly different from the coefficient for Real SPLT (.33868), bidders reveal that they are not using a conversion factor of 6.5 board feet per ton. The RSPLT variable is defined as REALSPLT divided by 6.5 for ton sales; otherwise it is 0 for board foot sales. The TONVOL variable is 1 for ton sales. Otherwise it is zero. In all sales (both ton and board foot, the board foot volume is included in the equation.

The west side model essentially decomposes to a ton equation and a board foot equation for each Westside appraisal zone.

For example, the NW zone equation for board foot sales is:

$$Y1 = 187.44 + 63.669 + .33693 * REALSPLT + 3.6151 * VOLSOLD \\ - 65.016 * LOGSYS \\ \text{or}$$

$$= 251.109 + .33693 * REALSPLT + 3.6151 * VOLSOLD - 65.016 * LOGSYS$$

For Ton Sales in the Northwest Zone, the equation is:

$$Y1 = 187.344 + .33693 REALSPLT - 0.66802 * REALSPLT / 6.5 + 3.6151 * VOLSOLD \\ - 0.31439 * TONVOL + (63.699 - 74.586) - 65.016 * LOGSYS + 40.350 * LOGSYS$$

This ton sale equation simplifies into the following

$$Y1 = 176.457 + .23412 REALSPLT + 3.6151 VOLSOLD - 0.31439 * TONVOL \\ - 24.666 * LOGSYS$$

While it would be redundant to do the same for each other zone in the Westside, the equations for the other zones would be deduced in the same way.

The extensive use of dummy variables makes it more difficult to present “understandable” equations. However, in order to use the most representative sample of sales over the entire appraisal period, it is necessary to keep the ton sales in the equation with the board foot sales. The estimation is far more tedious, but the result is a more representative set of appraisal equations and a better estimate of fair market value for the period.